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An Analysis of Students' Mathematical Reasoning in Solving Geometry Problems in Terms of Spatial Ability at SMP Islam Klojen Lumajang

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Abstract

Students' mathematical reasoning in solving geometry problems is significantly influenced by their spatial ability; however, an in-depth qualitative description of the relationship between these two variables remains scarce at the junior high school level. This study aims to describe the mathematical reasoning of junior high school students in solving flat-faced three-dimensional geometry problems, categorized by their spatial ability (high, moderate, and low). Employing a descriptive qualitative approach, the research involved eight ninth-grade students from SMP Islam Klojen Lumajang as subjects. Participants were selected using spatial ability and mathematical reasoning tests, followed by observations, in-depth interviews, and documentation. Data analysis was conducted through data reduction, data display, and conclusion drawing, with validity established via technique and source triangulation. The findings revealed a tiered pattern: students with low spatial ability exhibited mechanical-procedural reasoning; those with moderate ability demonstrated situational reasoning (proficient in concrete contexts but deficient in abstract problems); and students with high spatial ability displayed reflective and consistent reasoning across all indicators. In conclusion, a higher degree of spatial ability corresponds to a more mature mathematical reasoning profile. Consequently, geometry instruction should be hierarchically designed to accommodate students' varying levels of spatial ability.

Keywords geometry: spatial ability: mathematical reasoning

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INTRODUCTION

Mathematics education plays a strategic role in equipping students with logical and systematic thinking skills, one of which is through the development of mathematical reasoning, a mental activity aimed at drawing conclusions based on verified facts (Sumartini, 2015). This ability becomes crucial when students are faced with geometry problems, as the reasoning process is significantly influenced by spatial ability, defined as the mental capacity to visualize, understand, and manipulate geometric objects in space (Etmy et al., 2017).

The actual conditions at SMP Islam Klojen Lumajang indicate that when teachers explain the concept of cube nets, many students still struggle to imagine how a two-dimensional plane can be folded into a three-dimensional figure. When given practice problems regarding the surface area of a rectangular prism, the majority of students immediately look for the formula without attempting to represent the shape through drawing. As expressed by one student, "I cannot draw it, Sir," this indicates that the reasoning process halts due to weak mental visualization of geometric objects. A similar phenomenon is also evident in volume-related problems, where students have difficulty distinguishing the dimensions of length, width, and height based on the descriptions provided in word problems.

Ariati and Juandi (2022) emphasize that mathematical reasoning demands comprehension alongside a structured and coherent way of thinking, ensuring that students do not fall into the trap of mere formula memorization. Etmy et al. (2017) confirmed a significant relationship between spatial tendencies and students' mathematical reasoning abilities in solving geometry problems. Furthermore, students' cognitive development in understanding geometry can be mapped using the Van Hiele theory, which explains the stages of thinking from simple visualization to formal deduction (Muhassanah et al., 2014). In line with this, Linda et al. (2023) found an influence of spatial intelligence on students' mathematical reasoning abilities, with an average accuracy rate of 71%.

Previous studies have generally examined the relationship between spatial ability and mathematical reasoning quantitatively; however, an in-depth description of how this relationship manifests in students' thinking processes across each spatial ability category remains limited. This is particularly true for students with moderate spatial ability, whose reasoning profiles can be situational. This research gap forms the rationale for the current study. This research aims to describe the mathematical reasoning of junior high school students in solving flat-faced three-dimensional geometry problems viewed from their spatial abilities, categorized into high, moderate, and low. Consequently, this study intends to provide a concrete overview as well as a foundation for developing geometry instructional strategies that are more responsive to the diversity of students' cognitive abilities.

METHODS

This research employed a descriptive qualitative approach to enable the researcher to capture the phenomenon of students' thinking processes comprehensively and profoundly within a natural school environment (Sugiyono, 2022; Fiantika et al., 2022). The research was conducted at SMP Islam Klojen Lumajang, involving ninth-grade students during the odd semester of the 2025/2026 academic year. This selection was based on the consideration that flat-faced three-dimensional geometry (cubes and rectangular prisms) had been thoroughly studied at this educational level.

The research subjects were selected through a spatial ability test and a mathematical reasoning test, which were previously validated by a mathematics education lecturer and a mathematics teacher. The validation results fell into the highly feasible category (81 to 100%). Based on the score intervals for spatial ability (high at 16 to 20, moderate at 11 to 15, and low at 5 to 10) and mathematical reasoning (high at 30 to 40, moderate at 20 to 29, and low at 10 to 19), eight research subjects were selected to represent the three spatial ability categories, as presented in Table 1.

The data sources consisted of primary and secondary data. Primary data included the students' actions and verbal expressions obtained from their test results and in-depth interviews. Secondary data comprised mathematics grade records, student profiles, and discussions with the subject teacher. Data were collected using three primary techniques: non-participant observation of student activities during the geometry test, semi-structured interviews to trace the students' flow of thought behind their written answers, and documentation. The documentation included archives of the students' work, interview transcripts, and visual records gathered throughout the research.

The data analysis followed the interactive model of Miles and Huberman as adapted by Sugiyono (2022). This process included data reduction (categorizing test results and interview transcripts based on spatial ability categories), data display (constructing descriptive narratives that juxtapose interview excerpts with written responses), and conclusion drawing along with verification. The validity of the data was ensured through technique triangulation, which involved comparing the results of written tests, interviews, and observations, as well as source triangulation, which involved cross-verifying the findings from the students with additional information from the mathematics teacher.

FINDINGS AND DISCUSSION

Findings

Based on the spatial ability test and the mathematical reasoning test administered to the ninth-grade students, eight subjects were obtained and distributed across three spatial ability categories: low (S2, S3, S7, S8), moderate (S1, S6), and high (S4, S5), as presented in Table 1.

Tabel 1. Spatial Ability and Mathematical Reasoning Categories of the Research Subjects

Subject Code	Spatial Ability Score	Spatial Category	Mathematical Reasoning Score	Reasoning Category
S1	12	Moderate	26	Moderate
S2	8	Low	19	Low
S3	5	Low	17	Low
S4	20	High	40	High
S5	18	High	36	High
S6	11	Moderate	20	Moderate
S7	7	Low	15	Low
S8	4	Low	10	Low

Source: source description

Regarding the indicator of proposing conjectures, subjects in the low category tended to immediately write down the final result based on guesswork without a logical initial conjecture. Subjects in the moderate category were able to propose conjectures but lacked consistency between contextual and abstract problems, whereas subjects in the high category were able to propose accurate conjectures from the beginning without the aid of illustrations. In the mathematical manipulation indicator, low-category subjects applied formulas mechanically without considering the problem context. Moderate-category subjects were capable of creatively modifying formulas for contextual problems but made errors on abstract problems, while high-category subjects consistently applied appropriate mathematical manipulations to both types of problems.

For the indicator of constructing proofs and providing reasons, subjects in the low category tended to write the final answer without showing the calculation process. Subjects in the moderate category provided reasons based on concrete experiences but struggled with problems lacking real-world references, whereas subjects in the high category constructed coherent proofs by initially outlining the geometric properties. Regarding the indicator of drawing valid conclusions, low-category subjects formulated conclusions without a clear reasoning basis. Moderate-category subjects experienced errors due to confusing the volume and surface area formulas, while high-category subjects drew conclusions that were consistent with all preceding steps. For the indicator of verifying the validity of arguments, which was the least frequently fulfilled indicator, low-category subjects did not perform any evaluation at all. Moderate-category subjects only realized their errors after the task session was completed, whereas high-category subjects actively verified the validity of their arguments throughout the process.

In the aspect of spatial ability, a similar tiered pattern was also discovered across its five indicators (visualization, mental rotation, spatial relations, spatial perception, and spatial orientation). Subjects in the low category were unable to visualize three-dimensional objects without the assistance of illustrations and failed to perform the mental rotation of nets. Subjects in the moderate category could visualize objects through real-life experiences but encountered spatial-motor coordination obstacles when translating them into two-dimensional representations. Meanwhile, subjects in the high category were able to perform visualization and mental rotation instantaneously, for example, through a mental anchoring strategy, which involves locking one central square of a net as a base anchor before imaginatively folding the other sides.

Discussion

Mathematical Reasoning of Students with Low Spatial Ability

Subjects with low spatial ability (S2, S3, S7, S8) possessed mathematical reasoning profiles that also fell into the low category, a consistent pattern across all indicators of both spatial ability and mathematical reasoning. The fundamental weakness lies in the subjects' inability to perform visualization and mental rotation of geometric objects, as evident when

S2 was unable to imagine the remaining height of the bathtub walls not submerged in water, and when S3 merely guessed the positions of the base and lid labels on a cube net without the process of mental folding. This condition has direct implications for the subjects' mathematical reasoning, which tended to be mechanical-procedural: formulas were memorized and applied without considering the context of the problem, such as the forced application of the full rectangular prism surface area formula on the bathtub problem, which should have been modified since portions of the walls were not filled with water.

These findings confirm the research results of Etmy et al. (2017) that there is a significant relationship between spatial tendencies and students' mathematical reasoning abilities in solving geometry problems; a weakness in one aspect is typically followed by a weakness in the other. The condition of these low-category subjects aligns with the perspective of Ariati and Juandi (2022) that students trapped in mere formula memorization do not yet possess coherent mathematical reasoning because they are unable to connect conceptual understanding with the chosen solution steps. The subjects' inability to propose initial conjectures and verify the validity of their own arguments, as seen from the behavior of S2 who merely followed the numbers presented in the problem, demonstrates that subjects in this category are still at the visualization stage of thinking according to the Van Hiele theory, the stage where students only recognize geometric shapes as a whole without understanding their constituent properties (Muhassanah et al., 2014). In line with Linda et al. (2023), these obstacles, which appear on the surface as procedural errors, are actually rooted in the inability to visualize geometric objects in space. Therefore, subjects in this category require instructional interventions that focus on strengthening concrete visualization before being directed towards abstract reasoning, because formal reasoning cannot be built without an adequate spatial foundation.

Mathematical Reasoning of Students with Moderate Spatial Ability

Subjects with moderate spatial ability (S1, S6) exhibited situational mathematical reasoning profiles, appearing optimal when problems were linked to real-world contexts but declining significantly when faced with abstract geometry problems. This is clearly illustrated by the ability of S1 to creatively modify the rectangular prism formula based on their experience with a bathtub at home, specifically by calculating only two length sides and two width sides with a remaining height of 20 cm. However, almost simultaneously, S1 made a fundamental error by confusing the formulas for the volume and surface area of a cube in a problem lacking a concrete reference. A similar pattern was also evident in the spatial aspect of S6, who was able to mentally rotate and accurately fold a net into a cube but experienced spatial-motor coordination barriers when attempting to translate the rotation result into a two-dimensional sketch.

This phenomenon reinforces the findings of Agustin et al. (2023) that students with moderate ability often fail to draw logical conclusions even though their completion process is relatively accurate, indicating that procedural mastery alone is insufficient to guarantee valid reasoning without comprehensive conceptual understanding. Viewed from the perspective of Nike K. (2015), this instability indicates that the deductive and inductive reasoning of moderate-category subjects have not been fully integrated. The subjects were able to reason inductively from concrete experiences in the bathtub case but were not yet capable of applying formal deductive reasoning based purely on geometric properties in the plain cube case. This condition places the subjects in a transitional stage between visualization and analysis within the Van Hiele theory, where students have begun to recognize the properties of three-dimensional figures but are not yet able to apply them consistently across all forms of problem representations (Muhassanah et al., 2014). The retrieval error experienced by S1, namely the realization of a formula error that only emerged after the task session concluded, further indicates that the ability to verify the validity of arguments in moderate-category subjects remains reactive rather than actively reflective during the thinking process. Consequently, instructional reinforcement for this group should be directed toward enhancing metacognitive awareness and habituating the transfer of strategies from concrete contexts to abstract representations.

Mathematical Reasoning of Students with High Spatial Ability

Subjects with high spatial ability (S4, S5) demonstrated consistent cognitive maturity across all indicators in both spatial ability and mathematical reasoning aspects, unaffected by the presence or absence of concrete contexts in the problems. In the spatial aspect, subjects were able to perform visualization and mental rotation instantaneously without requiring the assistance of drawn sketches, as demonstrated by the mental anchoring strategy applied by S5, which involved locking one central square of a net as a base anchor before imaginatively folding the other sides. In the reasoning aspect, S4 was able to construct proofs systematically by initially outlining the geometric properties of a cube. For instance, the subject sequentially multiplied the area of one side of the cube (64 cm^2) by six congruent sides to obtain 384 cm^2 before arriving at a valid numerical conclusion. This indicates that their thinking process moved from conceptual understanding toward conclusion drawing, rather than the reverse.

This maturity aligns with the formal deduction thinking stage in the Van Hiele theory, where students are capable of constructing arguments based on geometric properties independently (Muhassanah et al., 2014). The high-category subjects' sensitivity to information traps within word problems, such as the precision of S4 and S5 in sorting crucial variables, specifically water height and unsubmerged areas, before performing calculations, also confirmed the achievement of the argument validity verification indicator. According to Setiyawan et al. (2024), this is the most difficult reasoning indicator for junior high school students to fulfill, even for those who have met the other four reasoning indicators. In other words, high-category subjects in this study had surpassed the mechanical-procedural mindset and moved toward a more comprehensive reflective reasoning. This simultaneously reinforces the view of Etmy et al. (2017) that good spatial ability correlates positively with more mature geometric reasoning, and it serves as empirical evidence that mastery of visualization and mental rotation is a crucial prerequisite for achieving the highest level of mathematical reasoning.

The Relationship Between Spatial Ability and Mathematical Reasoning

Overall, the findings of this study indicate a consistent relationship between the level of spatial ability and the mathematical reasoning profiles of students in solving flat-faced three-dimensional geometry problems. A higher degree of spatial ability possessed by a student corresponds to a more mature reasoning ability across the five investigated indicators, namely proposing conjectures, performing mathematical manipulations, constructing proofs and providing reasons, drawing valid conclusions, and verifying the validity of arguments. This tiered pattern is clearly evident, starting from low-category subjects who tended to be mechanical-procedural, moderate-category subjects who were situational, to high-category subjects who were reflective and consistent.

These results both reinforce and enrich the findings of Etmy et al. (2017) and Linda et al. (2023), who discovered that spatial intelligence influences students' mathematical reasoning abilities with an average accuracy rate of 71 percent (moderate effect category), thereby confirming the relationship between spatial ability and students' mathematical reasoning in three-dimensional geometry materials. The primary contribution of this research lies in the in-depth description of how this relationship manifests concretely in students' thinking processes, particularly among moderate-category subjects. These subjects demonstrated that spatial ability and mathematical reasoning do not always develop linearly and uniformly across all indicators but can instead be situational depending on the presence or absence of a concrete context in the encountered problems. These findings provide practical implications for mathematics teachers that the instruction of flat-faced three-dimensional geometry needs to be designed hierarchically: strengthening concrete visualization for students with low spatial ability, habituating the transfer of strategies from real-world contexts to abstract representations for students with moderate spatial ability, and providing non-routine problem enrichment that challenges the reflective sensitivity of students with high spatial ability. Consequently, the results of this study not only address the research focus regarding how junior high school students' mathematical reasoning is viewed from their spatial abilities but also provide a foundation for developing geometry instructional strategies that are more responsive to the diversity of students' cognitive abilities.

CONCLUSION

This research concludes that there is a close and tiered relationship between the spatial ability and mathematical reasoning of junior high school students in solving flat-faced three-dimensional geometry problems. Students with low spatial ability exhibited rote-procedural reasoning and were situated at the visualization stage according to the Van Hiele theory. Students with moderate spatial ability demonstrated situational reasoning, performing optimally in real-world contexts but declining on abstract problems, and were positioned at the visualization-analysis transitional stage. Meanwhile, students with high spatial ability demonstrated mature, reflective, and consistent reasoning across all indicators, having reached the formal deduction stage. Weak spatial ability constitutes the primary root of students' difficulties in solving geometry problems, rather than merely a lack of formula memorization.

Based on these conclusions, it is recommended that mathematics teachers design hierarchical and differentiated geometry instruction tailored to students' spatial ability levels, as well as routinely integrate spatial ability exercises into the learning process. Students are advised to actively practice their spatial abilities through activities such as drawing nets and connecting the learning materials to tangible objects, while schools are recommended to provide adequate geometric teaching aids. Future research is suggested to utilize a larger sample size or a mixed-methods approach to quantitatively test the significance of this relationship, as well as to develop geometry instructional models based on the reinforcement of spatial ability.

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